Author's Reply[2]

Stephan B. Worm

Ling Chen does not give any arguments why (2) should be like he suggests, but maybe the following explanation can make my definition more clear.

If we write the solution for a wave propagating in positive z-direction as

$$\begin{pmatrix} \psi_o^e \\ \psi_o^h \end{pmatrix}$$

then a solution for a wave in negative z-direction can be written as

$$\begin{pmatrix} -\psi_o^e \\ \psi_o^h \end{pmatrix}.$$

A general wave is now represented by

$$\begin{pmatrix} \psi^e \\ \psi^h \end{pmatrix} = A \begin{pmatrix} \psi_o^e \\ \psi_o^h \end{pmatrix} e^{-j\beta z} + B \begin{pmatrix} -\psi_o^e \\ \psi_o^h \end{pmatrix} e^{j\beta z}.$$

The potential function ψ^h has a z-dependency like E_y and thus like the voltage between a microstrip and ground. For the voltage reflection coefficient at $z = 0$ we obtain $r = B/A$.

With the discretization method we can impose a Dirichlet boundary condition for ψ^e and a Neumann condition for ψ^h (or reverse, but not twice the same condition):

$$\psi^e(z = 0) = A\psi_o^e + rA(-\psi_o^e) = (1 - r)A\psi_o^e.$$

With a normalization to $A = 1/(1 - r)$ we can use the solution ψ_o^e as obtained from the propagation problem at the boundary $z = 0$.

It then follows that

$$\partial\psi^h/\partial z \ (z = 0) = -j\beta\psi_o^h.$$

[2]Manuscript received June 17, 1991.
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IEEE Log Number 9103912.

Comments on "An Analytic Algorithm for Unbalanced Stripline Impedance"

Robert E. Canright, Jr.

Abstract—**This letter corroborates the results of recent research, promotes an alternative technique for calculating the impedance of unbalanced stripline, and highlights some older references that are often overlooked.**

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The author is with Martin Marietta Missile Systems, MP-200, P.O. Box 555837, Orlando, FL 32855-5837.
IEEE Log Number 9103910.
[1]P. Robrish, *IEEE Trans. Microwave Theory Tech.*, vol. 38, no. 8, pp. 1011-1016, Aug. 1990.

This author commends P. Robrish for solving the conformal mapping problem for the unbalanced stripline in the above paper.[1] Finding the easiest technique to calculate the characteristic impedance of unbalanced stripline is an important practical problem that has been under assault for some time [1]. Robrish's work fills an important hole in the theory. For the sake of completeness, someone needed to work through the conformal mapping, extending Cohn's [2] earlier work, and give us a solution for a stripline that is not centered between the ground planes. However, this author recommends a different technique that produces equivalent results, is easier to use, and has a flexibility that makes it more powerful [3]. First, the alternative will be presented, then Robrish's algorithm will be corroborated, then this letter will conclude with some discussion.

The alternate approach is very easy to state:

$$Z_0 = 2\, Z_{01} Z_{02}/(Z_{01} + Z_{02}) \tag{1}$$

where

Z_{01} = the stripline impedance based on the distance to the near ground plane

Z_{02} = the stripline impedance based on the distance to the far ground plane

as shown in Fig. 1.

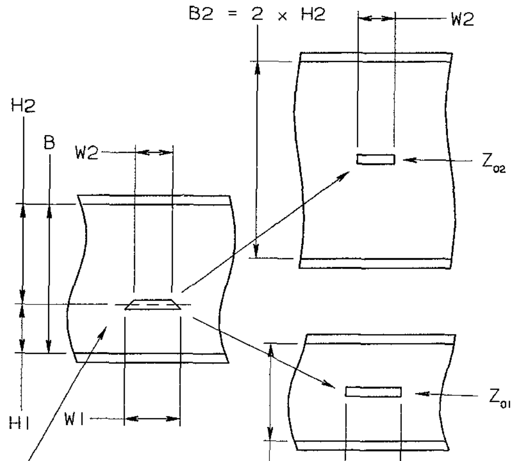
Looking at Fig. 1, the reader should notice that two line widths can be accommodated. This means that the sloping side walls that sometimes appear in printed wiring board (PWB) conductors can be accounted for when calculating the line impedance. Gupta [1] had looked for an easy algorithm to account for this effect, which occurs when the copper conductor is etched and is commonly called "undercut." Fig. 2 shows that conventional stripline impedance can also account for undercut effects by using this alternate technique. Undercut is an effect unaccounted for in Robrish's technique, which is why this author suggests that this new technique is more powerful. The phrase "new technique" means that the reader is expected to be unfamiliar with it, not that it is recent. This alternate technique was first presented without proof in 1987 [4], with its derivation [3] published later (April 1990).

Table I shows the comparison between Robrish's formulas and the use of (1) when analyzing six designs. Robrish's formulas were used to create the six designs, which is why the impedances have such tidy values when analyzed with the same formula used to create the design. To emphasize how the accuracy of (1) depends upon the stripline formulas used to calculate Z_{01} and Z_{02}, both Wheeler's technique [5] and Cohn's technique [2] were used for the analysis. The differences in impedance between Robrish's formulas and (1) are generally less than or equal to 1 percent, certainly within the 2 percent accuracy of Robrish's formulas as cited in the conclusion of his very fine paper. Hence, the results of (1), in conjunction with Cohn's technique, are essentially equivalent to Robrish's formulas.

Regarding Table I: B is the distance in mils between ground planes. The distance between the conductor and the near ground plane is $B/3$ ($cl = b/3$ using Robrish's terms). The conductor width, w, is 5.00 mil. The conductor thickness, t, is 1.4 mil. The PWB dielectric constant is 4.8. The impedance is in ohms.

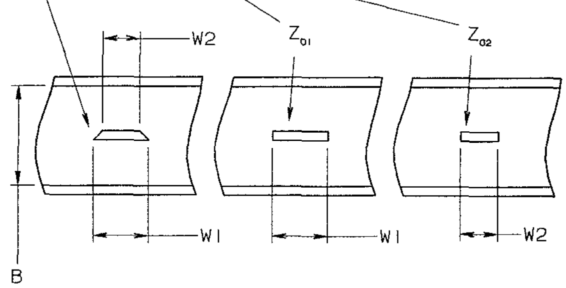
Cohn's and Robrish's techniques require a round wire approximation to the rectangular conductor. Robrish uses

$$D = (2/3)(0.8w + t). \tag{2}$$



$$Z_0 = \frac{2\, Z_{01}\, Z_{02}}{(Z_{01} + Z_{02})}$$

Fig. 1. Creating related stripline designs.



$$Z_0 = \frac{2\, Z_{01}\, Z_{02}}{(Z_{01} + Z_{02})}$$

Fig. 2. Accounting for undercut in conventional stripline.

TABLE I
A NUMERICAL COMPARISON OF IMPEDANCE FORMULAS

B	Z_0 Robrish	Z_0 Eq. (1) +Wheeler	Percent Difference	Z_0 Eq. (1) +Cohn	Percent Difference
13.86	40.00	40.72	1.82	40.30	0.77
20.27	50.00	50.88	1.75	50.37	0.74
24.33	55.00	55.83	1.51	54.44	−1.03
35.05	65.00	65.81	1.25	64.68	−0.50
50.49	75.00	75.85	1.13	74.86	−0.19
60.60	80.00	80.88	1.10	79.93	−0.09

The most commonly used formula [6] comes originally from Kaupp [7]:

$$D = 0.67w(0.8 + t/w) = 0.67(0.8w + t). \tag{3}$$

Kaupp's equation, however, is another form of Springfield's original equation [9]:

$$D = 0.567w + 0.67t. \tag{4}$$

The form of the equation used for the round wire approximation makes a small but perceptible difference in the calculations. The close agreement between Robrish's technique and (1) comes from using Springfield's equation (4) for D in conjunction with Cohn's technique and (1). Robrish's use of (2) does work better than either (3) or (4) in his formula.

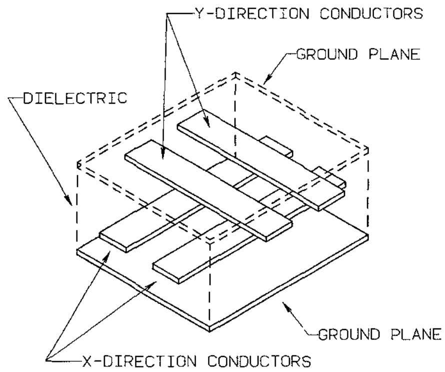


Fig. 3 Dual stripline has two layers of conductors between grounds.

Designing offset, or unbalanced, stripline becomes very easy when (1) is used. Already having written a stripline design program that calls an external subroutine for the calculation of impedance, one only needs in this case to make a small modification to a copy of the original stripline design program in order to have a program for offset stripline design. The modification takes about 15 min. Both design programs call the same subroutine for Z_0 calculation. Equation (1) can build upon and extend existing software.

Finally, it is worthwhile to discuss terminology. The term "unbalanced stripline" is an uncommon name for this conductor construction. The most common name is "offset stripline," in this author's opinion [9], [10]. The term "asymmetric stripline" is occasionally seen, but is most often used to describe a pair of coupled striplines having different widths [11]. "Triplate" has been used by persons from the IBM Corporation [1], [12], but triplate is an old term for stripline [13]. This author often uses the term "dual stripline" because it indicates the intention to use orthogonal pairs of conductor planes, as shown in Fig. 3, to increase the packaging density for high speed digital systems.

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Author's Reply[2]

P. Robrish

R. E. Canright's letter presents a rule of thumb for calculating the impedance of an unbalanced stripline. However, I think he oversells his case, and his remarks can lead the unwary reader into dangerous territory.

Since Canright's rule rests on a naive sort of superposition principle to calculate the capacitance of an asymmetric geometry, its generalization is certainly wrong. Therefore, it is incumbent on him to define, carefully, its limits of applicability. While Canright gives specific instances yielding errors of less than 2%, he has not pressed the model to find its limitations. For example, while it's true that for the line dimensions he mentions, the errors may be small for $cl/b = 1/3$, they rise to more than 6% at $cl/b = 1/5$, and are rapidly increasing with decreasing cl/b at that point. Canright's extension of his ideas to undercut lines offers the unsupported promise that a simple rule may obviate the need for a sound analytic treatment of this important problem. It's easy to conclude that this rule must fail, at least in some obvious limiting cases. For example, for a thin inner conductor whose upper face is much narrower than its lower face, Canright's rule will certainly underestimate the capacitance badly.

Finally, it's difficult to know how to respond to Canright's gratuitous comment about nomenclature other than to apologize to those whose linguistic sensibilities I might have offended but, in my defense, to point out that my terminology was sufficiently clear to the editor and referees.

[2]Manuscript received May 13, 1991.
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